Monitoring, Liquidation, and Security Design

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By identifying the possibility of imposing a credible threat of liquidation as the key role of informed (bank) finance in a moral hazard context, we characterize the circumstances under which a mixture of informed and uninformed (market) finance is optimal, and explain why bank debt is typically secured, senior, and tightly held. We also show that the effectiveness of mixed finance may be impaired by the possibility of collusion between the firms and their informed lenders, and that in the optimal renegotiation-proof contract informed debt capacity will be exhausted before appealing to supplementary uninformed finance.

This article develops a model of how entrepreneurial firms source their financing needs. There are three alternatives for raising finance: uninformed, informed, and a mixture of both. Under informed finance the lender observes at a certain cost the entrepreneur's level of effort, which determines the probability of success of his project. Although this information cannot be used to enforce a contingent contract, it enables the lender to liquidate the project (and recover part of the investment) if the observed effort does not guarantee her a sufficient continuation payoff. When liquidation values are large enough, a credible threat of liquidation leads the entrepreneur to choose first-

We would like to thank Leonardo Felli, Julian Franks, Paolo Fulghieri, Denis Gromb, John Moore, and David Webb for helpful comments and discussions. This article also greatly benefited from the comments of Franklin Allen (the editor) and two anonymous referees. Address correspondence to Rafael Repullo, CEMFI, Casado del Alisal 5, 28014 Madrid, Spain; e-mail: repullo@cemfi.es.

The Review of Financial Studies Spring 1998 Vol. 11, No. 1, pp. 163–187

best effort. Otherwise it is impossible to ensure a sufficiently tough liquidation policy without compromising the lender's participation constraint.

The conflict between preserving the credibility of the liquidation threat and compensating the lender provides a rationale for mixed finance: adding a passive uninformed lender allows a reduction in the funds contributed by the informed lender and hence restores the credibility of the threat. Our analysis shows that, for some entrepreneurs, mixed finance can improve upon both uninformed and informed finance. Thus it may explain why many firms are not exclusively funded by informed lenders (such as banks) or uninformed lenders (such as small bondholders), but by a mixture of both.

The effectiveness of mixed finance may be impaired by the possibility of collusion between the entrepreneur and the informed lender (to the detriment of the uninformed lender). In particular, if these informed parties can renegotiate their share of continuation proceeds after the effort decision has been made, first-best effort is no longer attainable. This renegotiation possibility determines the form of the optimal three-party contracts. Our results predict that, in order to give the informed lender the right incentives to liquidate, informed debt will be, in case of liquidation, secured and senior to uninformed debt. Moreover, in the optimal renegotiation-proof contract informed debt capacity (the maximum informed debt compatible with a credible liquidation threat) will always be exhausted.

We aim to offer a testable theory of the choice of the mix of informed and uninformed finance. Given the active role assigned to informed lenders under the optimal contracts, we will argue that private debt such as bank loans can be considered informed finance, whereas public debt such as corporate bonds or outside equity can be considered uninformed finance. We identify two key determinants of the optimal mode of finance: the level of entrepreneurial wealth (or the firm's net worth) and the liquidation value of the investment project. We predict that investments which involve nonspecific liquid and tangible assets are more likely to be funded exclusively by banks or large active investors, while as we move to projects involving less and less redeployable assets we will observe increasing reliance on arm's-length finance.

This article is related to the literature on debt contracts that has stressed the disciplinary role of liquidation. Hart and Moore (1989) and Bolton and Scharfstein (1990) consider models in which cash flows are unverifiable, showing that in this context it is optimal to give liquidation rights to the lenders in order to discourage strategic default. Berglöf and von Thadden (1994) analyze the rationale for multiple lenders in a similar setting. They show that if liquidation values are low, it is optimal that short-term and long-term claims be held by separate investors, and short-term claims be secured. This arrangement strengthens the ex post bargaining position of the short-term lenders and diminishes the firm's incentives to default strategically. Our article transmutes into a moral hazard context the insight that a second lender may be useful to ensure the credibility of liquidation threats.[1]

In Diamond (1993a, 1993b), short-term debt forces borrowers to renegotiate their contracts after some signal about their quality is publicly observed. So depending on the signal, firms are either liquidated or refinanced on terms more closely related to their actual prospects. The liquidation rights associated with short-term debt are important for dealing with the adverse selection problem. However, refinancing short-term debt entails a risk of inefficient liquidation due to the loss of control rents. He shows that introducing junior long-term debt and allowing the issue of additional senior debt in the future reduces this risk. In contrast to our article, the second lender comes in to prevent excessive liquidation rather than to restore the credibility of the liquidation threat. From an empirical viewpoint, Diamond highlights bank priority over cash flows in a context where control rents and refinancing risk are important, whereas we stress bank priority over liquidated assets when moral hazard problems are pervasive.

Finally, other articles derive implications for the design of the priority structure and covenants of different classes of debt in settings where the emphasis shifts from the discipline associated with liquidation threats to the idea that different classes of lenders have different abilities to renegotiate [Detragiache (1994)], different reputations for monitoring well [Chemmanur and Fulghieri (1994)], or different incentives to monitor [Rajan and Winton (1995)]. In most respects, their results and ours can be thought of as complementary.

This article is organized as follows. Section 1 describes the model. Sections 2 and 3 characterize the optimal contracts under uninformed and informed finance, respectively. Section 4 presents our results on mixed finance. Section 5 analyzes the optimal choice between these modes of finance. Section 6 contains a discussion of the implications of the model. Section 7 concludes.

1. The Model

Consider a model with four dates ($t = 0, 1, 2, 3$) and a continuum of risk-neutral entrepreneurs. Each entrepreneur has the opportunity of

[1] Rajan (1992) studies the trade-off between bank and arm's-length debt in a moral hazard model similar to ours. He examines the impact of bank lenders' ex post bargaining power on the efficiency of the entrepreneur's effort decision. He stresses the importance of the *hold-up* problem as a cost of bank debt, suggesting (unlike us) that arm's-length debt should have priority over bank debt. An alternative moral hazard setting is explored in Gorton and Kahn (1993).

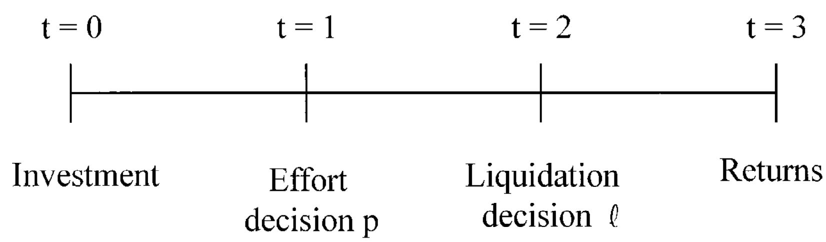

Figure 1
Timing of events

undertaking an *indivisible project* that requires an investment at $t = 0$ which is normalized to one. The entrepreneur can affect the outcome of the project through the amount of *costly effort*, $p \in [0, 1]$, expended at $t = 1$. At $t = 2$ the project can be liquidated. The indicator variable ℓ will take the value 1 if liquidation occurs and 0 otherwise. Contingent on p and ℓ, the project yields verifiable returns at $t = 3$. The timing of events is depicted in Figure 1.

If the project is undertaken and liquidation does not take place ($\ell = 0$), with probability p the project is successful and the return is $Y > 0$, whereas with probability $1 - p$ the project fails and the return is 0. If the project is liquidated ($\ell = 1$), a certain return $L > 0$ is obtained, irrespective of p. The cost of effort $\phi(p)$ is incurred regardless of the outcome of the project.

Each entrepreneur is characterized by his initial wealth w and the liquidation value of his project L. We will restrict attention to the case where $w < 1$, so entrepreneurs require external finance in order to undertake their projects.

We make the following assumptions.

Assumption 1. *The function $\phi(p)$ is increasing and strictly convex, and satisfies $\phi(0) = \phi'(0) = 0$ and $\lim_{p \to 1} \phi'(p) = +\infty$.*

Assumption 2. *There exists a perfectly elastic supply of funds at an expected rate of return which is normalized to zero.*

Assumption 3. $\max\{pY - \phi(p)\} \equiv \bar{p}Y - \phi(\bar{p}) > 1.$

Assumption 4. $L < 1.$

Assumption 5. *The entrepreneur's effort decision p is not contractible.*

Assumption 1 is standard and is made to ensure that the entrepreneur's maximization problem is convex and has a unique interior solution. Assumption 2 is used to close the model in a very simple manner, normalizing the expected rate of return required by lenders to zero. Assumption 3 (together with Assumption 2) ensures that the net present value of the project when the entrepreneur chooses the first-best level of effort $\bar{p}$ is positive. Notice that, by Assumption 1, $\bar{p} \in (0, 1)$ and is characterized by the first-order condition

$$Y = \phi'(\bar{p}), \tag{1}$$

which equates the marginal benefit of effort to its marginal cost. Assumption 4 (together with Assumption 2) states that investing in order to liquidate is not profitable. Finally, Assumption 5 introduces a moral hazard problem. In particular, if an entrepreneur with wealth w borrows $1 - w$ in exchange for a promise to repay $R \in [0, Y]$, and the project is never liquidated, he will choose p in order to maximize $p(Y - R) - \phi(p)$. The solution to this problem is characterized by the first-order condition

$$Y - R = \phi'(p), \tag{2}$$

which implicitly defines the entrepreneurial choice of p as a function of R. Comparing Equations (1) and (2), and using the assumption that $\phi'' > 0$, one obtains that the solution for p in Equation (2) is smaller than the first-best level of effort $\bar{p}$.

In what follows, we examine the disciplinary role of liquidation threats by lenders in this moral hazard setup. The relationship between an entrepreneur and his lender is assumed to be governed by a contract, signed at $t = 0$, that specifies how the parties agree to share the funding and the verifiable returns of the project under both liquidation and no liquidation.

Formally, a contract between an entrepreneur and a lender is described by a vector (I, Q, R) that specifies (i) the funds I invested by the lender in the project, (ii) the part Q of the liquidation proceeds L which go to the lender if she decides to liquidate, and (iii) the part R of the success return Y that is paid to the lender if she does not liquidate.

For expositional convenience, we will assume that each entrepreneur invests his entire wealth w in the project, showing later that this is indeed optimal. Sections 2 and 3 study the optimal contracts between lenders and entrepreneurs under uninformed and informed finance, respectively. Under uninformed finance, the choice of p by the entrepreneur is not only noncontractible but also unobservable to the lender. Under informed finance, a costly technology is used by the lender to observe p.

2. Uninformed Finance

Under uninformed finance, given a contract (I, Q, R), the interaction between an entrepreneur and his lender can be modeled as a game with imperfect information. In this game, the entrepreneur first chooses the level of effort $p \in [0, 1]$, and then the lender, without observing the entrepreneur's decision (thus the imperfect information), takes the liquidation decision $\ell \in \{0, 1\}$. The payoff to the entrepreneur is $L - Q - \phi(p)$ if the project is liquidated, and $p(Y - R) - \phi(p)$ otherwise. The payoff to the lender is Q if she liquidates the project and pR otherwise.

A contract (I, Q, R) for an entrepreneur with wealth w is said to be feasible under uninformed finance if there exists a (pure strategy) Nash equilibrium (p^*, ℓ^*) such that

$$(1 - \ell^*)p^*R + \ell^*Q \geq I = 1 - w, \tag{3}$$

and

$$(1 - \ell^*)p^*(Y - R) + \ell^*(L - Q) - \phi(p^*) \geq w. \tag{4}$$

Equations (3) and (4) are participation constraints for the lender and the entrepreneur, respectively. A feasible contract has to provide enough funds to undertake the project, it has to guarantee the lender the required expected rate of return, and it has to provide the entrepreneur with an expected utility greater than (or equal to) the value of his initial wealth.

A feasible contract (I, Q, R) for an entrepreneur with wealth w is said to be optimal under uninformed finance if it maximizes the equilibrium expected utility of the entrepreneur in the class of all feasible contracts.[2]

The following result characterizes the optimal contracts under uninformed finance.

Proposition 1. *There exists a critical value $\bar{w}_u \in [0, 1)$ such that, for any entrepreneur with wealth $w \geq \bar{w}_u$, the optimal contracts under*

[2] In the definitions of feasible and optimal contracts we have restricted attention to pure strategy equilibria. This is done without loss of generality, because allowing for mixed strategy equilibria does not change the sets of feasible and optimal contracts. To sketch why this is so, notice first that for any given probability of liquidation chosen by the lender, the payoff to the entrepreneur is strictly concave in the level of effort p, so the entrepreneur never mixes. Moreover, the value of p chosen by the entrepreneur is decreasing in the probability of liquidation. With regard to the lender, there might be equilibria in which she randomizes her choice of ℓ, while the participation constraints [Equations (3) and (4)] are satisfied. Nevertheless, liquidating with positive probability is inefficient, since it worsens the entrepreneur's incentives and (given $L < 1$) reduces the overall surplus. Therefore the mixed strategy equilibria associated with feasible contracts (if they exist) are always Pareto dominated by the unique pure strategy equilibrium of the game.

uninformed finance are given by

$$I_u(w) = 1 - w, \quad Q_u(w) \in [0, \min\{L, 1 - w\}], \text{ and}$$
$$R_u(w) = (1 - w)/p_u(w), \tag{5}$$

where $p_u(w)$ is the largest value of p that solves the equation

$$p[Y - \phi'(p)] = 1 - w. \tag{6}$$

For $w < \bar{w}_u$ there is no feasible contract under uninformed finance.

Proof. The Nash equilibrium (p^*, ℓ^*) of the game defined by any feasible contract has to satisfy $\ell^* = 0$; otherwise adding up the participation constraints [Equations (3) and (4)] we would get $L - \phi(p^*) \geq 1$, which contradicts Assumption 4. But then the threat of liquidation cannot play any role in the optimal contract. Since $\ell^* = 1$ and the definition of equilibrium implies $Q \leq p^* R$, we can pick any $Q \in [0, \min\{L, p^* R\}]$, and focus on the optimal choice of R.

This requires finding the best solution for the entrepreneur to the system of equations formed by the entrepreneur's first-order condition [Equation (2)] and the lender's (binding) participation constraint $pR = 1 - w$. Substituting $R = (1 - w)/p$ into Equation (2) gives the equation $f(p) \equiv p[Y - \phi'(p)] = 1 - w$. Under Assumption 1, the function $f(p)$ is continuous and satisfies $f(0) = f(\bar{p}) = 0$. Moreover, it is positive for $p \in (0, \bar{p})$ and negative for $p \in (\bar{p}, 1)$. Then it is clear that the equation $f(p) = 1 - w$ has at least one solution if $\hat{f} \equiv \max f(p) \geq 1 - w$, and any solution will be smaller than $\bar{p}$ (since by assumption $1 - w > 0$). Now substituting $pR = 1 - w$ into the entrepreneur's payoff function gives the function $U(w, p) \equiv w + pY - \phi(p) - 1$. Since $U(w, p)$ is increasing in p for $p < \bar{p}$, it follows that the value of p corresponding to the optimal contract is the largest solution $p_u(w)$ to the equation $f(p) = 1 - w$, and $R_u(w) = (1 - w)/p_u(w)$.[3]

The entrepreneur's participation constraint requires $V_u(w) \equiv U(w, p_u(w)) \geq w$. The function $p_u(w)$ is increasing, continuous from the right, and satisfies $\lim_{w \to 1} p_u(w) = p$. Since $U(w, p)$ is increasing in w and in p for $p < \bar{p}$, it follows that $V_u(w)$ is increasing and, by Assumption 3, satisfies $\lim_{w \to 1} V_u(w) > w$, so for large values of w the participation constraint will be satisfied. Now let $\hat{w} \equiv \max\{1 - \hat{f}, 0\}$. Then if $V_u(\hat{w}) \geq \hat{w}$, the critical value $\bar{w}_u$ is given by $\hat{w}$. If, on the other hand, $V_u(\hat{w}) < \hat{w}$, $\bar{w}_u$ is defined by the conditions $V_u(w) \geq w$ for $w \geq \bar{w}_u$, and $V_u(w) < w$ for $w < \bar{w}_u$. ∎

[3] It should be noticed that the first-order condition [Equation (2)] implies $Y - R_u(w) = \phi'(p_u(w)) > 0$, so the payment promised to the lender is always smaller than Y.

It should be noted that since $pY - \phi(p)$ is increasing in p for $p < \bar{p}$, and $p_u(w)$ is smaller than $\bar{p}$ and increasing in w, the equilibrium expected utility under uninformed finance $V_u(w) \equiv w + p_u(w)Y - \phi(p_u(w)) - 1$ satisfies $V_u(w + \varepsilon) > V_u(w) + \varepsilon$ for all $w \geq \bar{w}_u$ and $\varepsilon > 0$. Hence it is optimal for the entrepreneur to invest all his wealth in the project.

Proposition 1 shows that under uninformed finance the option to liquidate has no value to the lender. There exists a critical level of wealth $\bar{w}_u$ such that only those entrepreneurs with wealth above $\bar{w}_u$ are able to fund their projects (whatever their value of L). When $w \geq \bar{w}_u$ the optimal contract is characterized by a promised payment to the lender $R_u(w) = I_u(w)/p_u(w)$. The term $1/p_u(w)$ can be interpreted as a default premium. As w goes down (increasing the reliance on external financing) the moral hazard problem becomes more severe, and so the default premium rises until the cutoff point $\bar{w}_u$ is reached. For $w < \bar{w}_u$ the moral hazard problem is so severe that uninformed finance is not feasible.

3. Informed Finance

In this section we introduce an alternative mode of financing the investment projects, which will be called informed finance. Specifically, we assume that the lender can (contractually) commit to use a monitoring technology that, at a cost $c > 0$ per project, reveals to her the value of p chosen by the entrepreneur. By Assumption 5 this information cannot be included in the contract between the lender and the entrepreneur, but it may be useful to the lender when deciding on liquidation.

The assumption that the lender can commit to monitor the entrepreneur is restrictive but fairly standard [see, e.g., Diamond (1991) and Rajan (1992)]. The information obtained by the lender in this mode of finance can be interpreted as the result of a continuous close relationship with the borrower along which the entrepreneur exerts his effort under the surveillance of the lender. This may involve, for example, regular interviews with the firm's executives and main customers, visits to the firm's premises, as well as (in the case of informed bank finance) observing the movements in the firm's bank accounts.[4]

[4] In an attempt to endogenize the informed lender's monitoring decision (in the context of a simplified version of the model with two levels of effort), we came to the conclusion that, although many of our results are robust to this change, such a setup would make the intuition behind them less transparent. In particular, situations in which monitoring takes place are associated with equilibria in which both the entrepreneur and the (potentially) informed lender play mixed strategies. The lender is indifferent between monitoring and not monitoring precisely because the entrepreneur mixes between high and low effort in such a way that the expected gain to the lender upon detection of low effort exactly compensates the monitoring cost.

Under informed finance, given a contract (I, Q, R), the interaction between an entrepreneur and his lender can be modeled as a sequential game in which the entrepreneur first chooses the level of effort p, and then the lender, after observing the entrepreneur's decision, takes the liquidation decision ℓ. The payoffs to the entrepreneur and the lender are the same as those for the case of uninformed finance.

A contract (I, Q, R) for an entrepreneur with wealth w is said to be feasible under informed finance if there exists a subgame perfect equilibrium $(p^*, \ell^*(p))$ such that

$$[1 - \ell^*(p^*)]p^* R + \ell^*(p^*) Q \geq I = 1 - w + c, \tag{7}$$

and

$$[1 - \ell^*(p^*)]p^*(Y - R) + \ell^*(p^*)(L - Q) - \phi(p^*) \geq w. \tag{8}$$

This definition of feasibility differs from that corresponding to uninformed finance in two respects. First, given the different nature of the game—which becomes genuinely sequential when the lender is informed—it refers to subgame perfect instead of Nash equilibrium. Second, it includes the monitoring cost c in the right-hand side of the lender's participation constraint [Equation (7)]. An equilibrium strategy of the lender specifies not only her reaction to the equilibrium strategy of the entrepreneur, $\ell^*(p^*)$, but also her reaction to entrepreneurial decisions off the equilibrium path, $\ell^*(p)$ for all $p \neq p^*$. However, the definition of feasibility only takes into account the players' decisions on the equilibrium path. As under uninformed finance, a feasible contract has to guarantee the lender the required expected rate of return on her initial investment (now including the monitoring cost c), and it has to provide the entrepreneur with an expected utility greater than (or equal to) the value of his initial wealth.

In order to make informed finance feasible, we will strengthen Assumption 3 to

Assumption 3′. $\max\{pY - \phi(p)\} \equiv \bar{p}Y - \phi(\bar{p}) > 1 + c$.

A feasible contract (I, Q, R) for an entrepreneur with wealth w is said to be optimal under informed finance if it maximizes the equilibrium expected utility of the entrepreneur in the class of all feasible contracts.

The following proposition characterizes optimal contracts under informed finance when the sum of the initial wealth of the entrepreneur w and the liquidation value of his project L is sufficiently large.

Proposition 2. *For any entrepreneur with $w + L \geq 1 + c$, the optimal* *contract under informed finance is given by*

$$I_i(w) = Q_i(w) = 1 - w + c \text{ and } R_i(w) = (1 - w + c)/\bar{p}. \quad (9)$$

Proof. We first show that under this contract, a subgame perfect equilibrium of the game between the entrepreneur and the informed lender is given by

$$p^* = \bar{p} \text{ and } \ell^*(p) = \begin{cases} 0, & \text{if } p \geq \bar{p} \\ 1, & \text{otherwise.} \end{cases} \quad (10)$$

To prove this, note that if $p < \bar{p}$ we have $pR_i(w) = p(1 - w + c)/\bar{p} < Q_i(w)$, so the lender will choose $\ell^*(p) = 1$. On the other hand, if $p > \bar{p}$ by the same argument she will choose $\ell^*(p) = 0$. Finally, if $p = \bar{p}$ the lender is indifferent between $\ell = 0$ and $\ell = 1$. Setting $\ell^*(\bar{p}) = 0$, the entrepreneur will choose $p^* = \bar{p}$ in the first stage of the game. This is because, by Assumption 1 and the definition of $\bar{p}$, $p[Y - R_i(w)] - \phi(p)$ is decreasing in p for $p \geq \bar{p}$, and we have

$$\begin{aligned} L - Q_i(w) - \phi(p) \ &\leq \ L - Q_i(w) < w < w + \bar{p}Y - (1 + c) - \phi(\bar{p}) \\ &= \ \bar{p}[Y - R_i(w)] - \phi(\bar{p}). \end{aligned}$$

The first inequality follows from the fact that $\phi(p) \geq 0$, the second from Assumption 4 and the definition of $Q_i(w)$, and the third from Assumption 3′. Since the equilibrium payoff of the lender is $1 - w + c$, and the equilibrium payoff of the entrepreneur is greater than w, Equation (9) is feasible. To prove that it is optimal it suffices to note that the equilibrium expected utility of the entrepreneur coincides with the maximum that he could achieve in the first-best world in which p was verifiable (but the costs of the project were $1 + c$). ■

According to Proposition 2, informed finance leads to the first-best choice of effort for those entrepreneurs with wealth $w + L \geq 1 + c$. This reflects the nature of the disciplinary device that operates under informed finance: the threat of liquidation. When liquidation proceeds are greater than the funds invested by the lender in the project ($L \geq 1 - w + c$), a contract that triggers liquidation whenever the entrepreneur chooses $p < \bar{p}$ can be signed at $t = 0$. The threat of liquidation is credible because the contractual value of Q can be chosen large enough to give proper incentives to the lender. On the equilibrium path, however, liquidation does not take place.

By Proposition 2, the equilibrium expected utility under informed finance for an entrepreneur with $w + L \geq 1 + c$ is $V_i(w) \equiv w + \bar{p}Y - \phi(\bar{p}) - (1 + c)$. As the slope of this function is equal to 1, investing all his wealth in the project is weakly optimal (he should invest at least $1 + c - L$).

Next we consider what happens when $w+L < 1+c$. Since feasible contracts cannot lead to liquidation on the equilibrium path (otherwise adding up the participation constraints of Equations (7) and (8) we would contradict Assumption 4), it must be the case that $p^*R \geq 1 - w + c > L \geq Q$. From here it follows that p^* is strictly greater than the critical $\hat{p}$ that triggers liquidation (i.e., that solves $\hat{p}R = Q$), so liquidation threats are not effective. Given this, we can prove a result similar to Proposition 1 characterizing the optimal contracts under informed finance for entrepreneurs with $w + L < 1 + c$. Since in this case the lenders have to recover the monitoring cost c, it is clear that these contracts are dominated by the corresponding optimal contracts under uninformed finance.[5]

Summing up, under informed finance the lender observes the effort put by the entrepreneur at a certain cost. This information may be used by the lender to decide on the liquidation of the project, but this is not always valuable. The threat of liquidation is effective in disciplining entrepreneurs with $w+L \geq 1+c$. When this condition is not satisfied, the threat of liquidation cannot be credible, and so (given the monitoring cost) informed finance is dominated by uninformed finance.

4. Mixed Finance

In Sections 2 and 3 we analyzed the problem of designing optimal two-party contracts between lenders and entrepreneurs under informed and uninformed finance. Somewhat surprisingly, informed finance leads to the first-best level of effort $\bar{p}$ for those entrepreneurs with wealth $w + L \geq 1 + c$, whereas it does not allow an improvement compared to uninformed finance when $w + L < 1 + c$. The reason for this is that low values of L in relation to the funds $1 - w + c$ the informed lender has to invest in the project impede the effective use of the threat of liquidation. There is a conflict between providing the lender with incentives to liquidate if a deviation from $\bar{p}$ occurs (that is, setting Q and R such that $\bar{p}R = Q \leq L$) and compensating her for her investment in the project (that is, setting R such that $\bar{p}R \geq 1 - w + c$). If $w+L \geq 1+c$ there exist Q and R such that $\bar{p}R = 1-w+c = Q \leq L$; otherwise the liquidation threat cannot be binding, and the information acquired by the lender at a cost c is completely worthless.

[5] By the same argument as in the proof of Proposition 1, the effort chosen by the entrepreneur in the optimal contract under informed finance, $p_i(w)$, is the largest solution to the equation $f(p) \equiv p[Y - \phi'(p)] = 1 - w + c$. Using Equation (6) together with the properties of the function $f(p)$ it is immediate that $p_i(w) < p_u(w)$, which implies $V_i(w) < V_u(w)$.

The nature of this conflict provides a prima facie case for mixed finance, the coexistence of an informed active lender whose contribution to the project is reduced to a level which provides her the right incentives to liquidate (if the entrepreneur deviates from $\bar{p}$), and an uninformed passive lender who contributes the rest. Such a possibility is explored in this section.

Under mixed finance, the relationship between an entrepreneur and two lenders, one informed and another uninformed, is assumed to be governed by a contract, signed at $t = 0$, that specifies how the parties agree to share the funding and the verifiable returns of the project under both liquidation and no liquidation.

Formally, a contract between an entrepreneur, an informed, and an uninformed lender is a vector $(I_i, I_u, Q_i, Q_u, R_i, R_u)$ that specifies (i) the funds I_i and I_u invested in the project by the informed and the uninformed lender; (ii) the parts Q_i and Q_u of the liquidation proceeds L which go to the informed and the uninformed lender if the former decides to liquidate; and (iii) the parts R_i and R_u of the success return Y that are paid to the informed and the uninformed lender if the former does not liquidate.

In what follows, we first analyze the optimal three-party contracts in the absence of any renegotiation. These contracts are, however, not robust to the possibility of collusion (and renegotiation) between the entrepreneur and the informed lender at the date when the option to liquidate has to be exercised. The optimal renegotiation-proof contracts are then derived.

4.1 Mixed finance without renegotiation

For the same reasons as in the case of pure uninformed finance, under mixed finance the uninformed lender is a passive player in the game between the three parties to the contract. The interaction between the entrepreneur and the informed lender can then be modeled as a sequential game in which the entrepreneur first chooses the level of effort p, and then the informed lender takes the liquidation decision ℓ. The payoff to the entrepreneur is $L - Q_i - Q_u - \phi(p)$ if the project is liquidated, and $p(Y - R_i - R_u) - \phi(p)$ otherwise. The payoff to the informed lender is Q_i if she liquidates the project and pR_i otherwise. Finally, the payoff to the uninformed lender is Q_u if the project is liquidated and pR_u if it is not.

Our earlier definitions of feasible and optimal contracts can be easily extended to the mixed finance case, so for the sake of brevity we skip their formal statement.

For entrepreneurs with $w + L \geq 1 + c$, the equilibrium expected utility under informed finance is already at its highest possible level under mixed finance (corresponding to the first-best with costs $1 + c$).

For this reason we focus on the case of entrepreneurs with $w + L < 1 + c$. The following proposition characterizes the optimal contracts under mixed finance when there is no renegotiation at $t = 2$.

Proposition 3. *For any entrepreneur with $w + L < 1 + c$, there is a family of optimal contracts under mixed finance, parameterized by $x \in (0, L]$, which is given by*

$$\begin{aligned} I_i(w, x) &= Q_i(w, x) = x, \quad R_i(w, x) = x/\bar{p}, \\ I_u(w, x) &= (1 - w + c) - x, \quad Q_u(w, x) = L - x, \ \textit{and} \\ R_u(w, x) &= [(1 - w + c) - x]/\bar{p}. \end{aligned} \tag{11}$$

Proof. We first show that for any $x \in (0, L]$, a subgame perfect equilibrium of the game between the entrepreneur and the informed lender is given by Equation (10). To prove this, note that if $p < \bar{p}$ we have $pR_i(w, x) = px/\bar{p} < Q_i(w, x)$, so the lender will choose $\ell^*(p) = 1$. On the other hand, if $p > \bar{p}$ by the same argument she will choose $\ell^*(p) = 0$. Finally, if $p = \bar{p}$ the lender is indifferent between $\ell = 0$ and $\ell = 1$. Setting $\ell^*(\bar{p}) = 0$, the entrepreneur will choose $p^* = \bar{p}$ in the first stage of the game. This is because, by Assumption 1 and the definition of $\bar{p}$, $p[Y - R_i(w, x) - R_u(w, x)] - \phi(p)$ is decreasing in p for $p \geq \bar{p}$, and we have

$$\begin{aligned} L - Q_i(w, x) - Q_u(w, x) - \phi(p) &\leq 0 < w + \bar{p}Y - (1 + c) - \phi(\bar{p}) \\ &= \bar{p}[Y - R_i(w, x) - R_u(w, x)] \\ &\quad -\phi(\bar{p}). \end{aligned}$$

The first inequality follows from the definitions of $Q_i(w, x)$ and $Q_u(w, x)$ and the fact that $\phi(p) \geq 0$, and the second from Assumption 3′.[6] Moreover, the players' participation constraints are satisfied, so the family of contracts described in Equation (11) is feasible. To prove that they are optimal it suffices to note that the equilibrium expected utility of the entrepreneur coincides with the maximum that he could achieve in the first-best world in which p was verifiable (but the costs of the project were $1 + c$). ■

According to Proposition 3, mixed finance leads to the first-best choice of effort even for entrepreneurs with $w + L < 1 + c$. The explanation for this result is simple: the presence of an uninformed lender allows a reduction in the contribution of the informed lender to $I_i \leq L$, so we can set $\bar{p}R_i = Q_i = I_i$, thereby restoring her incentives

[6] Notice that $Q_u(w, x)$ could be chosen to be smaller than $L - x$, as long as the entrepreneur does not prefer liquidation to continuation (with $p = \bar{p}$), that is, provided that $L - x - Q_u(w, x) \leq w + \bar{p}Y - (1 + c) - \phi(\bar{p})$.

to liquidate if the entrepreneur deviates from $\bar{p}$ while compensating her for her investment in the project.

It is interesting to note that in these optimal three-party contracts the informed lender is fully secured in the case of liquidation, that is, $Q_i(w, x) = I_i(w, x)$, while the uninformed lender is not, that is, $Q_u(w, x) < I_u(w, x)$. This feature of the optimal contracts, which will be further discussed below, may be interpreted as the seniority of informed debt, which arises endogenously in order to restore the credibility of the liquidation threat.

4.2 The effects of renegotiation between the informed parties

The results obtained so far on mixed finance do not take into account the possibility of renegotiation between the entrepreneur and the informed lender after the former has made his effort decision but before the latter decides on liquidation. Given the presence of a third party (the uninformed lender), renegotiation in this context should be understood in terms of an additional contract between the two informed parties that changes the payment promised to the informed lender, if she does not liquidate the project, to R_i'.

The exclusion of the uninformed lender from this renegotiation is explained by the fact that she is not informed about p. This may seem restrictive since, with two informed agents (and no constraints on contractibility), it is generally possible to design a mechanism that truthfully reveals this information to a third, uninformed, agent. However, the introduction of such a mechanism is impeded by the noncontractibility of p (that is, the impossibility of describing the level of effort in a way suitable for enforcing contracts contingent upon it).[7]

In what follows we first show that the contracts described in Proposition 3 are not robust to renegotiation between the informed parties. We then characterize the optimal renegotiation-proof contracts under mixed finance.

In the renegotiation game, the status quo payoffs of the entrepreneur and the informed lender are $p(Y - R_i - R_u) - \phi(p)$ and pR_i, respectively, and in addition the lender has an *outside option* (the option to liquidate) which is worth Q_i to her. If $p(Y - R_u) < Q_i$, the informed lender would liquidate the project, since the maximum expected payment under continuation is smaller than what she can get

[7] If, nevertheless, the uninformed lender became informed and participated in the renegotiation, mixed finance would not improve on pure informed finance: assuming efficient renegotiation, the critical $\hat{p}$ that triggers liquidation would solve $\hat{p}Y = Q_i + Q_u \leq L$, whilst feasibility would require $p^*Y \geq I_i + I_u = 1 - w + c > L$. Hence p^* would be strictly greater than $\hat{p}$, so liquidation threats would not be effective.

upon liquidation. On the other hand, if $p(Y - R_u) \geq Q_i$, by the "outside option principle"[8] the equilibrium outcome of the renegotiation game is

$$R_i'(p) = \begin{cases} R_i, & \text{if } pR_i \geq Q_i \\ Q_i/p, & \text{otherwise.} \end{cases}$$

Thus the initial contract will be renegotiated if the probability of success p chosen by the entrepreneur satisfies $Q_i/(Y - R_u) \leq p < Q_i/R_i$,[9] in which case the informed lender's payoff $pR_i'(p)$ will be equal to her liquidation payoff Q_i. Anticipating this outcome, the entrepreneur will choose $p \geq Q_i/(Y - R_u)$ in order to maximize

$$p[Y - R_i'(p) - R_u] - \phi(p) = \begin{cases} p(Y - R_i - R_u) - \phi(p), & \text{if } pR_i \geq Q_i \\ p(Y - R_u) - \phi(p) - Q_i, & \text{otherwise.} \end{cases}$$

For the contract in Proposition 3, the condition $pR_i \geq Q_i$ reduces to $p \geq \bar{p}$. But then given that, by Assumption 1 and the definition of $\bar{p}$, $p[Y - R_i - R_u] - \phi(p)$ is decreasing in p for $p \geq \bar{p}$, and we also have $(Y - R_u) - \phi'(\bar{p}) < Y - \phi'(\bar{p}) = 0$, the entrepreneur has an incentive to choose $p^* < \bar{p}$ and subsequently bribe the informed lender in order to avoid liquidation. The uninformed lender will then get $p^* R_u < \bar{p} R_u = I_u$, so anticipating this outcome she will not be willing to participate in the funding of the project.[10]

Given this negative result, the following proposition characterizes the optimal renegotiation-proof contracts under mixed finance.

Proposition 4. *There exists a critical value $\bar{w}_m \in [\bar{w}_u, 1 + c)$ such that, for any entrepreneur with $w + L \in [\bar{w}_m, 1 + c)$, the optimal renegotiation-proof contract is given by*

$$\begin{aligned} I_i(w, L) &= Q_i(w, L) = L, \quad R_i(w, L) = L/p_m(w, L), \\ I_u(w, L) &= (1 - w + c) - L, \quad Q_u(w, L) = 0, \text{ and} \\ R_u(w, L) &= [(1 - w + c) - L]/p_m(w, L), \end{aligned} \tag{12}$$

[8] This principle is formulated in the context of a noncooperative bargaining model with alternating offers in which one of the players (say player 1) can quit the negotiations to take up an outside option [see Sutton (1986) and Osborne and Rubinstein (1990)]. In general, it states that if the value of this option is smaller than the equilibrium payoff of player 1 in the game with no outside option, then the option has no effect on the equilibrium outcome. Otherwise the equilibrium payoff of player 1 is equal to the value of his option.

[9] Note that $R_i + R_u \leq Y$ implies $Q_i/(Y - R_u) \leq Q_i/R_i$. Moreover, for the contracts described in Proposition 3, these inequalities are strict.

[10] Note that in the case of pure informed finance $R_u = 0$ implies $p^* = \bar{p}$, so the contract in Proposition 2 is robust to renegotiation.

where $p_m(w, L)$ *is the largest value of* p *that solves the equation*

$$p[Y - \phi'(p)] = 1 - w + c - L. \tag{13}$$

For $w + L < \bar{w}_m$ *there is no feasible contract under mixed finance.*

Proof. See the Appendix.

The result in Proposition 4 can be explained as follows. Mixed finance without renegotiation leads to liquidation by the informed lender if the entrepreneur deviates from the first-best level of effort $\bar{p}$. However, if we allow for renegotiation, entrepreneurial deviations are not necessarily followed by liquidation because the informed parties will bargain over the sharing of the continuation surplus: the liquidation threat enters as an outside option for the informed lender that provides a lower bound to her expected payoff. Since the uninformed lender is an outsider to this renegotiation, her stake will not be considered as a component of the expected continuation surplus to be bargained between the informed parties. By the "outside option principle," the equilibrium renegotiation payoffs of the entrepreneur and the informed lender will be $p(Y - R_u) - \phi(p) - Q_i$ and Q_i, respectively. Given this outcome, the solution to the entrepreneur's maximization problem will be a decreasing function of R_u that approaches the first-best level of effort $\bar{p}$ as R_u tends to zero. From here it follows that the entrepreneur will be interested in signing a contract in which the contribution I_u of the uninformed lender, and so the (irrevocable) payment R_u promised to her under continuation, are minimized.

Comparing Equations (6) and (13), we can see that the probability of success $p_m(w, L)$ chosen by the entrepreneur under this contract is equal to $p_u(w + L - c)$, that is the probability of success chosen by an entrepreneur with wealth $w + L - c$ under uninformed finance. This means that, under mixed finance, the liquidation value of the project plays the role of additional wealth that helps improve entrepreneurial incentives, since as noted in Section 2 the function $p_u(w)$ is increasing.

Using this result, the equilibrium expected utility under mixed finance for an entrepreneur with $w + L \in [\bar{w}_m, 1 + c)$ can be written as $V_m(w, L) \equiv w + p_u(w + L - c)Y - \phi(p_u(w + L - c)) - (1 + c)$. Hence, by our previous argument, it is again optimal for him to invest all his wealth in the project.

Two final comments are in place. First, although the possibility of collusion between the informed parties reduces the efficiency of mixed finance (given that $p_m(w, L) < \bar{p}$), some entrepreneurs can obtain funds that they could not get under pure informed or uninformed finance (in particular, those with $w + L \in [\bar{w}_m, 1 + c)$ and $w < \bar{w}_u$).

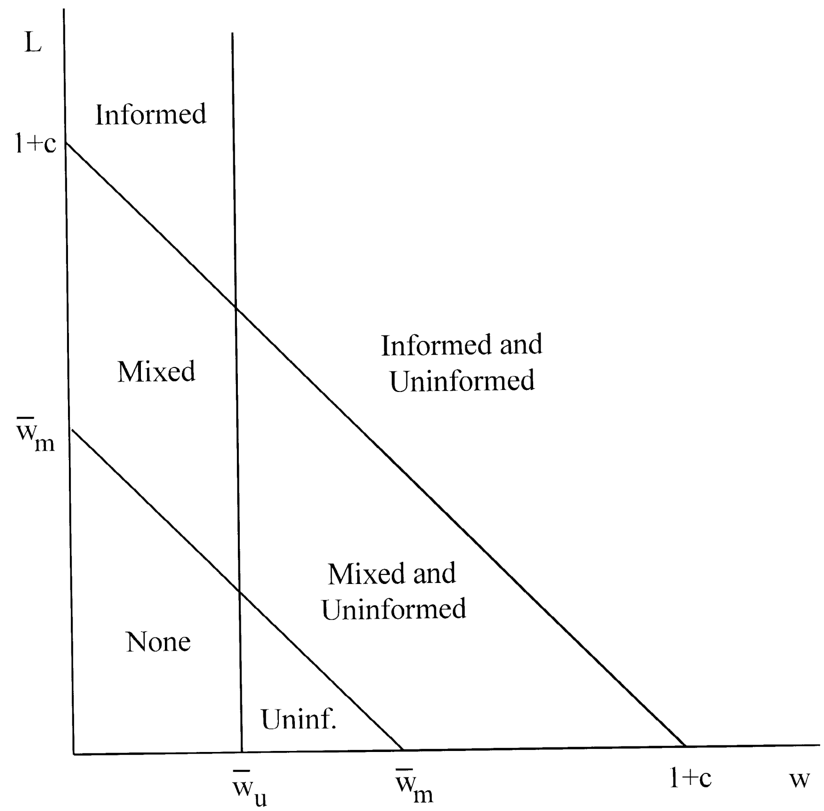

Figure 2
Feasible modes of finance
The entrepreneur's initial wealth (w) and the liquidation value of his project (L) determine the modes of finance that are feasible. The feasibility of uninformed finance only depends on w, whereas that of informed and mixed finance depends on the sum of w and L.

Second, in the optimal renegotiation-proof contract, informed debt is, in the case of liquidation, fully secured and senior to uninformed debt.

5. The Choice Between Informed, Uninformed, and Mixed Finance

This section brings together the results of the previous sections in order to analyze the optimal choice between informed, uninformed, and mixed finance. We begin by summarizing in Figure 2 our results on the regions of the $w - L$ space where these modes of finance are feasible.

According to Proposition 1, uninformed finance is feasible for all $w \geq \bar{w}_u$. By Proposition 2, informed finance is feasible for all pairs (w, L) above or on the line $w + L = 1 + c$; informed finance is also feasible for some pairs (w, L) below this line, but in these cases it is strictly dominated by uninformed finance. Finally, by Proposition 4, mixed finance (with renegotiation between the informed parties) is feasible for pairs (w, L) with $w + L \in [\bar{w}_m, 1 + c)$, where $\bar{w}_m \geq \bar{w}_u$.[11]

Since two modes of finance are feasible in two of the regions in Figure 2, we next consider which one dominates in each of them.

Proposition 5. *In the region where both uninformed and informed finance are feasible, there exists a unique $w^* \in [\bar{w}_u, 1)$ such that the former dominates the latter for $w \geq w^*$.*

Proof. The entrepreneurs' equilibrium expected utility under uninformed finance is $V_u(w) \equiv w + p_u(w)Y - \phi(p_u(w)) - 1$, whereas his expected utility under informed finance is $V_i(w) \equiv w + pY - \phi(\bar{p}) - (1 + c)$. Since $\lim_{w \to 1} p_u(w) = \bar{p}$, we have $\lim_{w \to 1}[V_u(w) - V_i(w)] = c > 0$. But then using the fact that $p_u(w)Y - \phi(p_u(w))$ is increasing in w, the result follows. ■

It is immediate to show that the critical value w^* is decreasing in the monitoring cost c, reaching the value $\bar{w}_u$ for large c.

Proposition 6. *In the region where both uninformed and mixed finance are feasible, there exists a function $L(w) \in [\max\{\bar{w}_m - w, 0\}, 1 - w + c]$ such that the former dominates the latter for those pairs (w, L) with $L < L(w)$. Moreover, $L(w) = 1 - w + c$ for $w \geq w^*$.*

Proof. The entrepreneur's equilibrium expected utility under mixed finance is $V_m(w, L) \equiv w + p_u(w + L - c)Y - \phi(p_u(w + L - c)) - (1 + c)$. Given that $\lim_{L \to 1-w+c} p_u(w + L - c) = \bar{p}$, we have $\lim_{L \to 1-w+c} V_m(w, L) = V_i(w)$. But by the definition of w^* in Proposition 5 we have $V_i(w) \leq V_u(w)$ if and only if $w \geq w^*$. Since $V_m(w, L)$ is increasing in L, the result follows. ■

Figure 3 summarizes our results on the characterization of the optimal modes of finance. Informed finance is optimal for high liquidation values and low entrepreneurial wealth. Uninformed finance is optimal for either high wealth or intermediate wealth and low liquidation values. Mixed finance is optimal for low entrepreneurial wealth and intermediate liquidation values. Finally, no mode of finance is feasible for low wealth and low liquidation values.

[11] This inequality is strict except in the limiting case where $\bar{w}_m = \bar{w}_u = 0$.

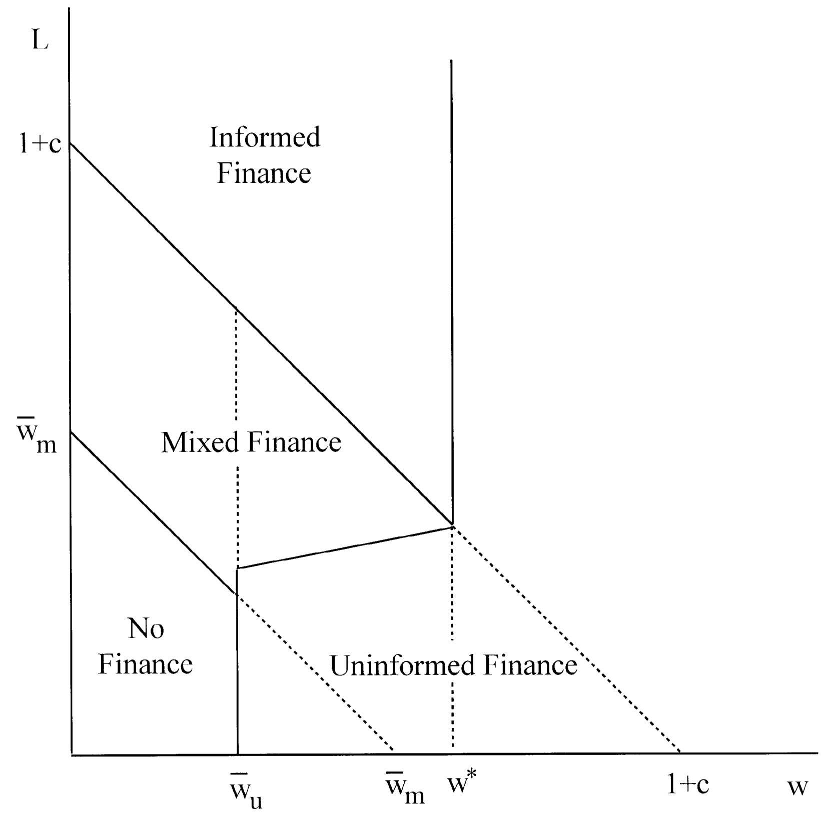

Figure 3
Optimal modes of finance
This figure shows the modes of finance that are optimal for different values of the entrepreneur's initial wealth (w) and the liquidation value of his project (L). In the region where informed and uninformed finance are both feasible, decreasing w worsens the moral hazard problem under uninformed finance, but does not affect informed finance, so informed finance is optimal for low w. In the region where mixed and uninformed finance are both feasible, decreasing w and increasing L so as to keep $w + L$ constant worsens the moral hazard problem under uninformed finance but does not affect mixed finance, so mixed finance is optimal for low w and high L.

Finally, we comment on the behavior of equilibrium interest rates for the different regions of Figure 3. According to Proposition 1, in the region where uninformed finance is optimal, equilibrium interest rates $R_u(w)/(1 - w) = 1/p_u(w)$ are decreasing in the level of entrepreneurial wealth w, because reducing the external financing requirement ameliorates the moral hazard problem. In the limit when w tends to 1, this problem disappears, and $1/p_u(w)$ approaches the value $1/\bar{p}$. By the reasons explained in Section 3, in the region where informed finance is optimal, the threat of liquidation eliminates the moral hazard problem, so equilibrium interest rates are constant and equal to $1/\bar{p}$. Finally, in the region where mixed finance is optimal, the moral hazard problem reappears due to the possibility of collusion between the informed parties. By Proposition 4, equilibrium interest rates $1/p_m(w, L)$ are in this case decreasing in both the level of entrepreneurial wealth w and the liquidation value of the project L. Moreover, $1/p_m(w, L)$ tends to $1/\bar{p}$ as $w + L$ approaches the value $1 + c$.

6. Discussion

The need for active monitoring under informed and mixed finance suggests the desirability of assigning this task to a single informed lender. It will save on the cost of monitoring (avoiding duplication) and will eliminate potential free-rider problems as well as conflicts over the exercise of the liquidation option. On the contrary, the passive role of uninformed lenders in uninformed or mixed finance can be performed by one or multiple lenders. These differences provide a rationale for identifying uninformed finance with the placing of publicly traded securities in the market (arm's-length finance) and informed finance with either bank lending or the issuance of tightly held (private) securities.[12]

With this interpretation, our model offers an explanation of the characteristics and coexistence of financial contracts such as typical bank loans and corporate bonds. In particular, our characterization of the securities associated with informed and uninformed finance, respectively, seems broadly consistent with the description of these contracts made by Gorton and Kahn (1993, p. 1): "A typical bank loan contract with a firm involves a single lender who is a secured senior debt claimant on the firm. The contract contains a large number of covenants which effectively give the lender the right to force the borrower to repay the loan early if demanded. In contrast, corporate bonds typically involve multiple lenders who are not secured, may not be senior, have less detailed covenants, and have no option to force the borrower to repay."

There are various real-world counterparts of the liquidation option which characterize our optimal contracts under both informed and mixed finance. If the entrepreneur's bargaining power is large, the optimal contract under informed finance can be approximately imple-

[12] Whether informed finance can be identified with *intermediated finance* is a question beyond the scope of this article: further specification of the relative sizes of investors' financial resources and entrepreneurs' financial needs, the stochastic dependence of the returns of the different investment projects, and the nature of the intermediaries would be required to address this issue.

mented by a sequence of short-term contracts.[13] Similarly, the optimal contract under mixed finance can be approximately implemented by a sequence of short-term contracts (with the informed lender) plus a long-term contract (with the uninformed lender), with the interesting property that short-term claims would be secured (and effectively senior to long-term claims) if they were not rolled over and subsequently the firm went into liquidation. Therefore, having the entrepreneur tied up with securities that mature before the project yields sufficient cash flows may be a way of granting an informed lender the option to liquidate. A similar effect could be achieved if the project were (totally or partially) financed (as commonly done by banks) through a line of credit callable at the option of the lender under “materially adverse circumstances”: these vaguely specified circumstances would correspond, in terms of our model, to the observation of an unsatisfactory level of effort.

In order to derive the empirical implications of the results summarized in Figure 3, we can associate the variable w with the firm’s net worth (relative to the size of its investment opportunities) and the variable L with some measure of the redeployable value of the investments. Then, among highly capitalized firms we would expect to observe a preference for the use of arm’s-length securities, such as public debt or outside equity. In contrast, banks or large active security holders would have a prominent role among poorly capitalized firms: either as the only financiers (for high liquidation values) or in conjunction with some form of arm’s-length finance (for lower liquidation values). The richest variety of modes of finance would be observed for firms in the middle range of net worth values. Among them, investments which involve nonspecific liquid and tangible assets (for example, those in basic industrial activities) would be funded exclusively by banks or large active investors (informed finance). As we move to projects involving more and more specific illiquid or intangible assets (for example, those in high-tech and service activities) we would observe increasing (and finally total) reliance on arm’s-length finance.

Some of these predictions are consistent with recent empirical findings. In particular, Alderson and Betker (1995) analyze a survey of firms reorganized under Chapter 11 for which there is information on the liquidation value of their assets. They show that firms in the lowest quartile of the distribution of liquidation costs (inversely related to our variable L) have a postbankruptcy financial structure with an

[13] If the entrepreneur had all the bargaining power [as, for example, in Berglof and von Thadden (1994)], the two alternatives would be equivalent.

average ratio of private debt to total debt of 0.816, whereas for those in the highest quartile (low *L*) the ratio is 0.531. Similarly, the average ratios of secured debt to total debt for the same groups of firms are 0.845 and 0.630, respectively.

From our results concerning the characteristics of contracts under each mode of finance, we would expect a lower dispersion of the implicit default premia for firms that borrow exclusively from banks (pure informed finance) than for firms that borrow from both banks and the market (mixed finance) or exclusively from the market (pure uninformed finance). We would also expect that a credit rating agency involved in assessing the quality of a public issue of corporate bonds would focus on the valuation of the firm's net worth rather than the specificity or liquidity of the assets involved in the new investments. In contrast, a bank or a large corporate lender would also pay attention to the redeployable value of the investments, trying to ensure that the threat of "pulling the plug" is effective.

7. Conclusion

This article discusses optimal security design in the context of a model of entrepreneurial firms' financing. We consider three alternatives for raising finance: uninformed, informed, and a mixture of both. We show that the key role of informed finance is to impose a credible threat of liquidation. However, the credibility of this threat fails when liquidation values are low, in which case a mixture of informed and uninformed finance may be optimal, and informed debt will be secured and senior to uninformed debt.

We argue that uninformed finance may be identified with the placing of publicly traded securities in the market and informed finance with either bank lending or the issuance of tightly held securities. With this interpretation, the model provides a number of predictions on the influence of observable variables, such as the firms' net worth and assets' liquidity, on the choice between modes of finance. Our results suggest the desirability of multivariate approaches in empirical studies on firms' financing decisions.

Appendix

Proof of Proposition 4. The optimal contract under mixed finance with renegotiation between the informed parties is a solution to the problem:

$$\max_{(I_i, I_u, Q_i, Q_u, R_i, R_u)} [\hat{p}(Y - R_u) - \max\{\hat{p}R_i, Q_i\} - \phi(\hat{p})] \tag{14}$$

subject to the constraints

$$\hat{p} \equiv \underset{p \geq Q_i/(Y-R_u)}{\operatorname{argmax}} \; [p(Y - R_u) - \max\{pR_i, Q_i\} - \phi(p)], \tag{15}$$

$$I_i + I_u = 1 - w + c, \quad Q_i + Q_u \leq L, \quad R_i + R_u \leq Y, \tag{16}$$

$$\max\{\hat{p}R_i, Q_i\} = I_i, \quad \hat{p}R_u = I_u,^{14} \tag{17}$$

and

$$\hat{p}(Y - R_u) - \max\{\hat{p}R_i, Q_i\} - \phi(\hat{p}) \geq w. \tag{18}$$

Let $\bar{w}_m$ be defined as $\bar{w}_u$ in Proposition 1, but for a case in which the lender's participation constraint is $pR = 1 - w + c$. To prove the result we first show that if $w + L \in [\bar{w}_m, 1 + c)$ the contract stated in the proposition satisfies Equations (15)–(18). By construction, $p_m(w, L)[Y - \phi'(p_m(w, L))] = 1 - w + c - L = p_m(w, L)R_u(w, L)$, which implies $[Y - R_u(w, L)] - \phi'(p_m(w, L)) = 0$, so $p_m(w, L) = \operatorname{argmax}[p(Y - R_u(w, L)) - \phi(p)]$. But since $p_m(w, L)R_i(w, L) = Q_i(w, L)$, we also have

$$p_m(w, L) = \operatorname{argmax}[p(Y - R_u(w, L)) - \max\{pR_i(w, L), Q_i(w, L)\} - \phi(p)].$$

Now, by construction, $p_m(w, L)[Y - R_i(w, L) - R_u(w, L)] = p_m(w, L)Y - (1 - w + c)$, and if $w + L \geq \bar{w}_m$ we have $p_m(w, L)Y - (1 - w + c) \geq w + \phi(p_m(w, L))$, so we conclude

$$p_m(w, L)[Y - R_i(w, L) - R_u(w, L)] \geq w + \phi(p_m(w, L)) > 0. \tag{19}$$

This implies $p_m(w, L)[Y - R_u(w, L)] > p_m(w, L)R_i(w, L) = Q_i(w, L)$, so $p_m(w, L) > Q_i(w, L)/[Y - R_u(w, L)]$, and the proposed contract satisfies Equation (15). As for the other constraints, they are either trivially satisfied or follow immediately from Equation (19).

Next consider an arbitrary contract $(I_i, I_u, Q_i, Q_u, R_i, R_u)$ for an entrepreneur with wealth w that satisfies the constraints of Equations (15)–(18). We are going to prove that this contract is dominated by the contract stated in the proposition. Substituting Equation (17) into Equation (14), and using the constraint $I_i + I_u = 1 - w + c$, it suffices to show that $w + L \geq \bar{w}_m$ and

$$p_m(w, L)Y - \phi(p_m(w, L)) \geq \hat{p}Y - \phi(\hat{p}). \tag{20}$$

For this, we first note that since the function in Equation (15) is concave (because $\phi''(p) > 0$ and for $p = Q_i/R_i$ we have $(Y - R_u) - \phi'(p) > (Y - R_i - R_u) - \phi'(p)$), and $Q_i/(Y - R_u) \leq Q_i/R_i$ (because $R_i + R_u \leq Y$),

[14] We are assuming, without loss of generality, that the lenders' participation constraints are satisfied with equality.

$\hat{p}$ must satisfy one of the following conditions: (i) $\hat{p} \geq Q_i/R_i$ and $(Y - R_i - R_u) - \phi'(\hat{p}) = 0$; (ii) $\hat{p} = Q_i/R_i$, $(Y - R_u) - \phi'(\hat{p}) > 0$ and $(Y - R_i - R_u) - \phi'(\hat{p}) < 0$; or (iii) $\hat{p} \leq Q_i/R_i$ and $(Y - R_u) - \phi'(\hat{p}) \leq 0$ (with strict inequality only if $\hat{p} = Q_i/(Y - R_u)$).

If $\hat{p}$ satisfies condition (i), then using Equations (16) and (17) we can write $[Y - (1-w+c)/\hat{p}] - \phi'(\hat{p}) = 0$, that is $\hat{p}[Y - \phi'(\hat{p})] = 1 - w + c$. But then using the properties of the function $f(p) \equiv p[Y - \phi'(p)]$, noted in the proof of Proposition 1, together with the definition of $p_m(w, L)$, we conclude that $w > \bar{w}_i$ and $\hat{p} < p_m(w, L) < \bar{p}$. But since $pY - \phi(p)$ is increasing for $p < \bar{p}$, this implies that Equation (20) holds.

Suppose next that $\hat{p}$ satisfies condition (ii). Then using Equations (16) and (17) we have $(Y - I_u/\hat{p}) - \phi'(\hat{p}) > 0$ and $[Y - (1-w+c)/\hat{p}] - \phi'(\hat{p}) < 0$, which implies $I_u < \hat{p}[Y - \phi'(\hat{p})] < 1 - w + c$. Moreover, $I_i = Q_i < L$ together with Equation (16) implies $1 - w + c - L \leq I_u$. Hence we have $1 - w + c - L < \hat{p}[Y - \phi'(\hat{p})] < 1 - w + c$. But then by the properties of the function $f(p)$ and the definition of $p_m(w, L)$, we conclude that $\hat{p} < p_m(w, L) < \bar{p}$ and $w > \bar{w}_m$, so Equation (20) also holds.

Finally if $\hat{p}$ satisfies condition (iii), we first note that if $\hat{p} = Q_i/(Y - R_u)$, then using the fact that $Q_i/(Y - R_u) \leq Q_i/R_i$ we would have

$$\hat{p}(Y - R_u) - \max\{\hat{p}R_i, Q_i\} = \hat{p}(Y - R_u) - Q_i = 0,$$

which contradicts Equation (18). Hence it must be $(Y - R_u) - \phi'(p) = 0$, so using Equation (17) we have $(Y - I_u/\bar{p}) - \phi'(\hat{p}) = 0$, that is $\hat{p}[Y - \phi'(\hat{p})] = I_u$. Moreover, $I_i = Q_i < L$ together with Equation (16) implies $1 - w + c - L \leq I_u < 1 - w + c$. Hence we have $1 - w + c - L \leq \hat{p}[Y - \phi'(\hat{p})] < 1 - w + c$. But then by the properties of the function $f(p)$ and the definition of $p_m(w, L)$ it must be the case that $\hat{p} \leq p_m(w, L) < \bar{p}$ and $w \geq \bar{w}_m$, so Equation (20) holds. ■

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